

Please write clearly in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 1

Thursday 25 May 2023

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
TOTAL	



Answer **all** questions in the spaces provided.

- 1** Find the number of solutions of the equation $\tanh x = \cosh x$

Circle your answer.

[1 mark]

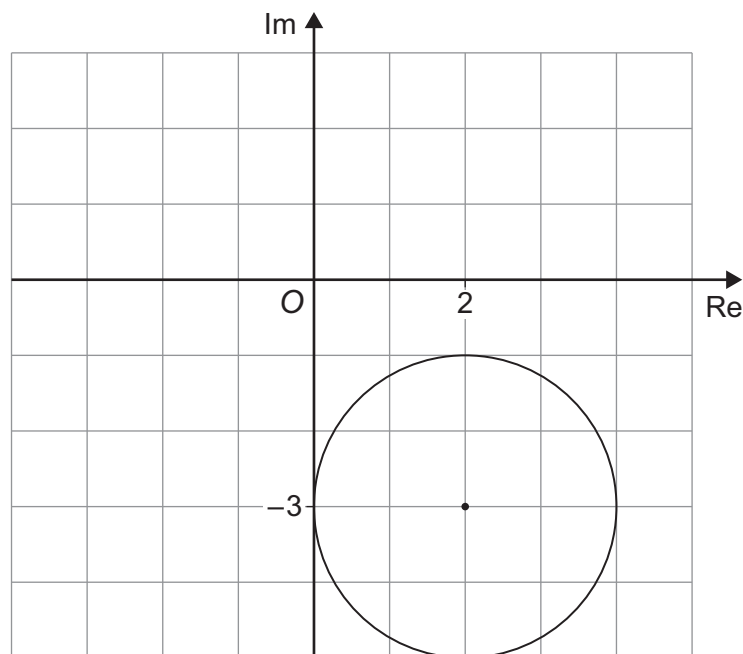
0

1

2

3

- 2** The diagram below shows a locus on an Argand diagram.



Which of the equations below represents the locus shown above?

Circle your answer.

[1 mark]

$|z - 2 + 3i| = 2$

$|z + 2 - 3i| = 2$

$|z - 2 + 3i| = 4$

$|z + 2 - 3i| = 4$



3

The matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ represents a transformation.

Which one of the points below is an invariant point under this transformation?

Circle your answer.

[1 mark]

(1, 1)

(0, 2)

(3, 0)

(2, 1)

4

The solution of a second order differential equation is $f(t)$

The differential equation models heavy damping.

Which one of the statements below could be true?

Tick (✓) **one** box.

[1 mark]

$$f(t) = 2e^{-t} \cos(3t) + 5e^{-t} \sin(3t)$$

☐

$$f(t) = 3e^{-t} + 4te^{-t}$$

☐

$$f(t) = 7e^{-t} + 2e^{-2t}$$

☐

$$f(t) = 8e^{-t} \cos(3t - 0.1)$$

☐

Turn over for the next question

Turn over ►



5 The function f is defined by

$$f(r) = 2^r(r - 2) \quad (r \in \mathbb{Z})$$

5 (a) Show that

$$f(r + 1) - f(r) = r2^r$$

[2 marks]



5 (b) Use the method of differences to show that

$$\sum_{r=1}^n r2^r = 2^{n+1}(n-1) + 2$$

[4 marks]

Turn over for the next question

Turn over ►



6 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \frac{1}{10} \begin{bmatrix} a & a & -6 \\ 0 & 10 & 0 \\ 9 & 14 & -13 \end{bmatrix}$$

where a is a real number.

The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are eigenvectors of $\mathbf{M}$

The corresponding eigenvalues are λ_1 , λ_2 , and λ_3 respectively.

It is given that $\lambda_2 = 1$ and $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} c \\ 0 \\ 1 \end{bmatrix}$,

where c is an integer.

6 (a) (i) Find the value of λ_1

[2 marks]

6 (a) (ii) Find the value of a

[2 marks]



[4 marks]

[illegible]

[3 marks]

[illegible]

$$f(x) = \left| \sin x + \frac{1}{2} \right| \quad (0 \leq x \leq 2\pi)$$
$$f(x) \geq \frac{1}{2}$$

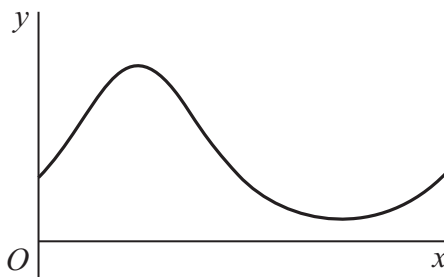
[5 marks]

[illegible]

8 The function g is defined by

$$g(x) = e^{\sin x} \quad (0 \leq x \leq 2\pi)$$

The diagram below shows the graph of $y = g(x)$



8 (a) Find the x -coordinate of each of the stationary points of the graph of $y = g(x)$, giving your answers in exact form.

[1 mark]

8 (b) Use Simpson's rule with 3 ordinates to estimate

$$\int_0^{\pi} g(x) \, dx$$

giving your answer to two decimal places.

[3 marks]

8 (c) Explain how Simpson's rule could be used to find a more accurate estimate of the integral in part **(b)**.

[1 mark]

Turn over ►



9 The position vectors of the points A , B and C are

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

$$\mathbf{b} = -\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{c} = -2\mathbf{j}$$

respectively.

9 (a) Find the area of the triangle ABC

[4 marks]



9 (b) The points A , B and C all lie in the plane Π

Find an equation of the plane Π , in the form $\mathbf{r} \cdot \mathbf{n} = d$

[2 marks]

9 (c) The point P has position vector $\mathbf{p} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

Find the exact distance of P from Π

[3 marks]

Turn over for the next question

Turn over ►



10 The matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & -1 & -2 \\ 1 & 2 & c \end{bmatrix}$$

where c is a real number.

10 (a) The linear transformation T is represented by the matrix **M**

Show that, for one particular value of c , the image under T of every point lies in the plane

$$x + 5y + 3z = 0$$

State the value of c for which this occurs.

[3 marks]

10 (b) It is given that **M** is a non-singular matrix.

10 (b) (i) State any restrictions on the value of c

[2 marks]



[4 marks]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.
$$x + 2y + 4z = 13$$

[3 marks]

[illegible]

11 The function f is defined by

$$f(x) = 4x^3 - 8x^2 - 51x - 45 \quad (x \in \mathbb{R})$$

11 (a) (i) Fully factorise $f(x)$

[2 marks]

11 (a) (ii) Hence, solve the inequality $f(x) < 0$

[2 marks]



11 (b) The graph of $y = f(x)$ is translated by the vector $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

The new graph is then reflected in the x -axis, to give the graph of $y = g(x)$

Solve the inequality $g(x) \leq 0$

[3 marks]

Turn over for the next question

Turn over ►



12 (a) Starting from the identities for $\sinh 2x$ and $\cosh 2x$, prove the identity

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

[2 marks]

12 (b) (i) The function f is defined by

$$f(x) = \tanh x \quad (x > 0)$$

State the range of f

[1 mark]

12 (b) (ii) Use part (a) and part (b)(i) to prove that $\tanh 2x > \tanh x$ if $x > 0$

[3 marks]



Use l'Hôpital's rule to prove that

$$\lim_{x \rightarrow \pi} \left(\frac{x \sin 2x}{\cos\left(\frac{x}{2}\right)} \right) = -4\pi$$

[5 marks]

[illegible]

Turn over for the next question

Turn over ►



14

14 (a) Show that r takes values in the range $\frac{1}{k} \leq r \leq k$, where k is an integer.

14 (a)

[2 marks]

14 (b)

[4 marks]

[illegible]

14 (c) The ellipse E has equation

$$y^2 + \frac{16x^2}{25} = 1$$

Find the transformation that maps the graph of E onto C

[4 marks]

Turn over for the next question

Turn over ►



Find the general solution of the differential equation

[9 marks]

[illegible]

[illegible]

16 (a) Show that

$$\int_{0.5}^4 \frac{1}{t} \ln t \, dt = a(\ln 2)^2$$

where a is a rational number to be found.

[4 marks]



16 (b)

$$x = 2t \quad y = \frac{1}{2}t^2 - \ln t$$

The arc formed by the graph of C from $t = 0.5$ to $t = 4$ is rotated through 2π radians about the x -axis to generate a surface with area S

Find the exact value of S , giving your answer in the form

$$S = \pi(b + c \ln 2 + d(\ln 2)^2)$$

where b , c and d are rational numbers to be found.

[7 marks]

[illegible]

Turn over ►



This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

END OF QUESTIONS



There are no questions printed on this page

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



Question number	Additional page, if required. Write the question numbers in the left-hand margin.
	Copyright information For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk. Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team. Copyright © 2023 AQA and its licensors. All rights reserved.

